Absolute Value

1. How far apart on a number line are
   a. 12 and 18?   b. 12 and -7?   c. -11 and -4?

2. What is the distance between 6 and -6? Between 24 and 17? Between 17 and 24?
   Between t and 4? The distance between two points is always positive. If a and b are two
   points on a number line, the distance is either $a - b$ or $b - a$, whichever is nonnegative.
   This is an example of an *absolute value* calculation, and the result is written $|a - b|$.
   What is the meaning of $|a - b|$?

3. On a number line, how far is each of the following numbers from zero?
   a. 45   b. -7   c. x d. $x + 2$ e. 0

4. On a number line, how far is each of the following numbers from 5?
   a. 17   b. -4   c. x d. $x + 3$ e. $x - 1$

5. Interpret each of the following as the distance between two numbers on a number line.
   a. $|x - 7|$ b. $|3 - x|$ c. $|x + 5|$ d. $|x|$

6. Find coordinates on a number line that are:
   a. six units from 0   b. six units from 4; c. six units from -7 d. six units from x

7. Rearrange the words "the, 4, between, 17, is, x, and, distance" to form a sentence that is
   equivalent to the equation $|x - 17| = 4$. Find the values of x by solving the equation.

8. Translate the sentence "the distance between x and 12 is 20" into an equation using
   algebraic symbols. What are the values of x being described?

9. Translate "x is 12 units from 20" into an equation. What are the values of x being
   described?

10. Translate the sentence "x and y are twelve units apart" into algebraic code. Find a pair
    (x,y) that fits this description. How many pairs are there?

11. The equation $|x - 7| = 2$ is a translation of "the distance from x to 7 is 2."
    a. Translate $|x - 7| \le 2$ into English, and graph its solution on a number line.
    b. Convert "the distance from -5 to x is at most 3" into symbol form, and solve it.

12. After successfully solving an absolute value problem, Robin spilled Heath Bar Crunch all
    over the problem. All that can be read now is "The distance between x and (mess of ice
    cream) is (another mess of ice cream)." Given that Robin's answers are $x = -3$ and $x = 7$,
    reconstruct the missing parts of the problem.

13. The specifications for machining a piece of metal state that it must be 12cm long, within a 0.01-cm tolerance. What is the longest the piece is allowed to be? What is the shortest? Using l to represent the length of the finished piece, write an absolute-value inequality that states these conditions.

14. If $|x + 1| = 5$, then $x + 1$ can have two possible values, 5 and -5. This leads to two equations, $x + 1 = 5$ and $x + 1 = -5$. If $|2x - 7| = 5$, what possible values could the expression $2x - 7$ have? Write two equations using the expression $2x - 7$ and solve them.

15. Write two equations without the absolute value symbols that, in combination, are equivalent to $|3x + 5| = 12$. Solve each of these two equations.

16. Fill in the blanks:
    a. The inequality $|x - 1.96| < 1.04$ is equivalent to " x is between ___ and ___."
    b. The inequality $|x - 2.45| \geq 4.5$ is equivalent to "x is not between ___ and ___."

17. Graph $y = |x - 5|$ and $y = |x + 3|$, then describe in general terms how the graph of $y = |x|$ is transformed to produce the graph of $y = |x - h|$.

18. Sketch on the same axes the graphs of $y = |x|$ and $y = |x| - 2$. Label the x- and y-intercepts. In what respects are the two graphs similar? In what respects do they differ?

19. Graph $y = |x| + 3$ and $y = |x| - 5$, then describe in general terms how the graph of $y = |x|$ is transformed to produce the graph of $y = |x| + k$. How can you tell from the graph whether k is positive or negative?

20. Ali phones Laura about a homework question, and asks, "The vertex of the graph of y equals the absolute value of x plus four is (-4,0), isn't it? Laura answers, "No, the vertex is (0,4)." Who is right? Explain.

21. Sketch on the same axes the graphs of
    a. $y = |x|$ b. $y = 2|x|$ c. $y = 0.5|x|$ d. $y = -3|x|$

22. What effect does the coefficient a have on the graph of the equation $y = a|x|$? How can you tell whether a is positive or negative by looking at the graph?

23. The fuel efficiency of a car depends on the speed at which it is driven. For example, consider Dani's Volvo. When it is driven at r miles per hour, it gets $m = 32 - 0.2|r - 55|$ miles per gallon. Graph m versus r, for $0 < r \leq 80$. Notice that this graph has a vertex. What are its coordinates?

24. Solve the inequality $30 \leq 32 - 0.2|r - 55|$, and express the solutions using *interval notation*. What is the meaning of these r-values to Dani?

25. Asked to solve the inequality $3 < |x - 5|$ at the board, Gabe wrote "$8 < x < 2$," Alex wrote "$x < 2$ or $8 < x$," and Stephen wrote "$x < 2$ and $8 < x$." What do you think of these answers? Do any of them agree with your answer?

26. Compare the graphs of $y = x - 3$ and $y = |x - 3|$. How are they related?

27. Felicity's way to solve the equation $|2x - 7| = 5$ is to first write $|x - 3.5| = 2.5$. Explain this approach and then finish the job.

28. Sandy was told by a friend that "absolute value makes everything positive." So Sandy wrote the equation $|x - 6| = x + 6$. Do you agree with this statement? Explain your answer.

29. For each of these absolute value equations, write two equations without absolute value symbols that are equivalent to the original. Solve them.
    (a) $2|x + 7| = 12$ (b) $3 + |2x + 5| = 17$ (c) $6 - |x + 2| = 3$ (d) $-2|4 - 3x| = -14$

30. The solution of $|x| = 6$ consists of the points 6 and -6. Show how to use a test point on the number line to solve and graph the inequality $|x| \le 6$. Do the same for $|x| \ge 6$.

31. What are the x- and y-intercepts of $y = |x - h| + k$, and what are the coordinates of its vertex?

32. Graph solutions on a number line:
       a. $|x + 8| < 20$ b. $|2x - 5| \le 7$ c. $3|4 - x| \ge 12$

33. Graph $y = 3|x - 2| - 6$, and find coordinates for the vertex and the x- and y-intercepts.

34. Your company makes spindles for the space shuttle. NASA specifies that the length of a spindle must be 12.45 ± 0.01 cm. What does this mean? What are the smallest and largest acceptable lengths for these spindles? Write these ranges of values as an inequality, letting L stand for the length of the spindle. Write another inequality using absolute values that models these constraints.

35. The fuel efficiency m (in miles per gallon) of a truck depends on the speed r (in miles per hour) at which it is driven. The relationship between m and r usually takes the form $m = a|r - h| + k$. For Sasha's truck, the optimal fuel efficiency is 24 miles per gallon, attained when the truck is driven at 50 miles per hour. When Sasha drives at 60 miles per hour, however the fuel efficiency drops to only 20 miles per gallon.

36.
   a. Find another driving speed r for which the fuel efficiency drops to only 20 mpg.
   b. Copy the table and fill in the missing entries.
   c. Draw the graph of m versus r, for $0 < r \le 80$.
   d. Find the values of k, a, and h.

r	m
60	20
50	24
40	
30	
20	
10	